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Angular Hydromagnetic Stability of Incompressible Dusty Fluid Flow between Two Rotating Cylinders

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Abstract: In this Paper we have examined the stability of inviscid, incompressible, dusty fluid between two co-axial rotating cylinders with different angular velocities in the presence of angular magnetic field. We found out the sufficient conditions for stability when $DN_0 < 0$ and $DN_0 > 0$ and observe that the effect of magnetic field is to stabilize. Also we obtained modes for non oscillatory and observe that the oscillatory modes are stable.

Keywords: ρ is density, μ_0 is magnetic permeability, perturbation, τ is Relaxation time, k^* is Stoke's resistance coefficient, μ is viscosity of clean fluid, π is pressure, N' is number density of dust particle.

1 INTRODUCTION

The necessary and sufficient condition of stability on physical grounds had been given by Rayleigh in 1916 and later on in 1938, Synge [6] gave the analytical proof of Rayleigh's criterion of stability. The problem of linearized stability of a plane parallel flow of dusty gas had been studied by Saffman [5] in 1962. A number of physical situations associated with the motion of dusty fluid between two rotating cylinders have been discussed by Greenpan (1983), Ungarish [7] (1988) and the stability thereof by Reeta in 1991.

Many research workers have done the work in last few years on the stability problems related to Couette flow in different physical contexts. Liu et.al (2001) examined the stability of an azimuthal base flow to both axisymmetric and plane-polar distributions for an electrically conducting fluid confined between stationary, concentric, infinitely long circular cylinder. An electric potential difference exists between the two cylinder walls and drives a radial electric current and without a magnetic field , this flow remains stationary. However, if an axial magnetic field is applied, the interaction between the radial electric current and the magnetic field give rise to an azimuthal electromagnetic body force which drives an azimuthal velocity. Infinitesimal axisymmetric disturbances lead to the instability in the base flow. Infinitesimal plane-polar disturbances do not appear to destabilize the base flow until shear flow transition to turbulence.

The effect of a rotating magnetic field on the stability of a fluid contained in a cylindrical column and heated from below was investigated by Volz and Mazurug [8] (1999). Dris [1] (1998) examined both experimentally and theoretically the flow of an elastic fluid between eccentric cylinders. They investigated the way the characteristics of the base flow ultimately influence the flow stability.

In this paper, we wish to examine the stability of inviscid, incompressible, dusty fluid between two co-axial rotating cylinders with different angular velocities in the presence of a magnetic field which is applied in the angular direction.

Following assumptions have been made to simplify the equations of motion:-

(a) The gap between the cylinders is small as compared to their radii.
(b) The perturbations are axisymmetric in nature.
(c) Relaxation time τ is small.
(d) Number density depends upon r
(e) Velocity of sedimentation is negligible.

2 FORMULATION OF THE PROBLEM

The initial state of which we wish to examine the stability or instability is given by

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$$\left.\begin{aligned}
\left(u_r, u_\theta, u_z\right) &= \left(0, U(r), 0\right)\\
\left(v_r, v_\theta, v_z\right) &= \left(0, U(r), 0\right)\\
\left(H_r, H_\theta, H_z\right) &= \left(0, H(constant), 0\right)\\
\rho = \rho_0(r), \quad & and \quad \pi = \pi_0(r)
\end{aligned}\right\} \tag{1}$$

provided that the equations $\frac{\partial \pi_0}{\partial r} = \frac{\rho_0 U^2}{r} - \frac{H^2}{\mu_0 r}$ and $g(r) = U^2 - \frac{H^2}{mN_0\mu_0}$ hold, where μ_0 stands for magnetic permeability.

Let $\rho', \pi', N', (u'_r, u'_\theta, u'_z), (v'_r, v'_\theta, v'_z)$ and (h'_r, h'_θ, h'_z) denote the perturbation in density, pressure, number density of dust particles, velocity of clear fluid, velocity of dust particle and the magnetic field respectively. Then the linearized hydromagnetic perturbation equations of the fluid particle layer are:

$$\rho_0\left[\frac{\partial u'_r}{\partial t} + \frac{U}{r}\frac{\partial u'_r}{\partial \theta} - 2\frac{Uu'_\theta}{r}\right] - \frac{\rho U^2}{r}$$
$$= -\frac{\partial \pi'}{\partial r} + k^* N_0\left(v'_r - u'_r\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_r}{\partial \theta} - \frac{2Hh'_\theta}{r}\right] \tag{2}$$

$$\rho_0\left[\frac{\partial u'_\theta}{\partial t} + u'_r\frac{\partial U}{\partial r} + \frac{U}{r}\frac{\partial u'_\theta}{\partial \theta} + u'_r\frac{U}{r}\right]$$
$$= -\frac{1}{r}\frac{\partial \pi'}{\partial \theta} + k^* N_0\left(v'_\theta - u'_\theta\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_\theta}{\partial \theta} + h'_r\frac{H}{r}\right] \tag{3}$$

$$\rho_0\left[\frac{\partial u'_z}{\partial t} + \frac{U}{r}\frac{\partial u'_z}{\partial \theta}\right]$$
$$= -\frac{\partial \pi'}{\partial z} + k^* N_0\left(v'_z - u'_z\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_z}{\partial \theta}\right] \tag{4}$$

$$\frac{\partial u'_r}{\partial r} + \frac{u'_r}{r} + \frac{1}{r}\frac{\partial u'_\theta}{\partial \theta} + \frac{\partial u'_z}{\partial z} = 0 \tag{5}$$

$$\frac{\partial \rho'}{\partial t} + u'_r\frac{\partial \rho_0}{\partial r} + \frac{U}{r}\frac{\partial \rho'}{\partial \theta} = 0 \tag{6}$$

$$mN_0\left[\frac{\partial v'_r}{\partial t} + \frac{U}{r}\frac{\partial v'_r}{\partial \theta} - \frac{2U}{r}v'_\theta\right]$$
$$= -mN'g + K^* N_0\left(u'_r - v'_r\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_r}{\partial \theta} - \frac{2Hh'_\theta}{r}\right] \tag{7}$$

$$mN_0\left[\frac{\partial v'_\theta}{\partial t} + v'_r\frac{\partial U}{\partial r} + \frac{U}{r}\frac{\partial v'_\theta}{\partial \theta} + \frac{U}{r}v'_r\right]$$
$$= K^* N_0\left(u'_\theta - v'_\theta\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_\theta}{\partial \theta} + h'_r\frac{H}{r}\right] \tag{8}$$

$$mN_0\left[\frac{\partial v'_z}{\partial t} + \frac{U}{r}\frac{\partial v'_z}{\partial \theta}\right]$$
$$= K^* N_0\left(u'_z - v'_z\right) + \frac{1}{\mu_0}\left[\frac{H}{r}\frac{\partial h'_z}{\partial \theta}\right] \tag{9}$$

$$\frac{\partial v'_r}{\partial r} + \frac{v'_r}{r} + \frac{1}{r}\frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_z}{\partial z} = 0 \tag{10}$$

$$\frac{\partial N'}{\partial t} + v'_r\frac{\partial N_0}{\partial r} + \frac{U}{r}\frac{\partial N'}{\partial \theta} = 0 \tag{11}$$

$$\frac{\partial h'_r}{\partial r} + \frac{h'_r}{r} + \frac{1}{r}\frac{\partial h'_\theta}{\partial \theta} + \frac{\partial h'_z}{\partial z} = 0 \tag{12}$$

$$\frac{\partial h'_r}{\partial t} + \frac{U}{r}\frac{\partial h'_r}{\partial \theta} - \frac{H}{r}\frac{\partial u'_r}{\partial \theta} = 0 \tag{13}$$

$$\frac{\partial h'_\theta}{\partial t} + \frac{U}{r}\frac{\partial h'_\theta}{\partial \theta} - \frac{H}{r}\frac{\partial u'_\theta}{\partial \theta} - h'_r\frac{\partial U}{\partial r} + \frac{U}{r}h'_r - \frac{H}{r}u_r = 0 \tag{14}$$

$$\frac{\partial h'_z}{\partial t} + \frac{U}{r}\frac{\partial h'_z}{\partial \theta} - \frac{H}{r}\frac{\partial u'_z}{\partial \theta} = 0 \tag{15}$$

Where $k^* = 6\pi\mu a$ is the stoke's resistance coefficient, a is the radius of dust particles, π is pressure and μ is viscosity of clean fluid.

To examine the stability of the stationary solution (1), we consider the infinitesimal axisymmetric perturbation of the form

$$f(r)\exp\left[i(pt + kz)\right] \tag{16}$$

where k is real wave number in the axial direction and p is complex frequency. Further if the fluid motion takes place between two coaxial vertical cylinders at r = R_1 and R_2 (> R1), then μ_r must vanish on the walls of the cylinders. The boundary conditions of the problem are thus given by

$$\mu_r = 0 \text{ at } r = R_1 \text{ and } R_2$$

after simplifying, we get a dispersion relation

$$pD\left[Du_r + \frac{u_r}{r}\right] + \frac{k^2}{p}\phi(r)u_r$$
$$-k^2 p\left[1 + \frac{k^*(DN_0)g\tau}{\rho_0 p\left(p + i\frac{DN_0}{N_0}g\tau\right)} - \frac{N^2}{p^2}\right]u_r + \frac{2\Omega_A^2}{pr^2}u_r = 0 \tag{17}$$

where $\Omega_A^2 = \frac{H^2k^2}{\mu_0\rho_0}$, $N^2 = \frac{D\rho_0}{\rho_0}\frac{U^2}{r}$

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and $\phi(r) = 2\Omega\left[\Omega + \frac{d}{dr}(r\Omega)\right]$ is the Rayleigh discriminant.

(17) is to be considered along with the boundary conditions

$$u_r = 0 \quad \text{at} \quad r = R_1 \quad \text{and} \quad R_2 (> R_1)$$

3 SUFFICIENT CONDITION FOR STABILITY

(a) Recovery of Rayleigh's criterion when $DN_0<0$

Multiplying (17) by ru_r^* and neglecting over the range of r and taking the imaginary part and then we get

$$p_i\left[I_1 + I_2 + I_3 + I_4\right] - I_5 = 0 \qquad (18)$$

where

$$I_1 = \int r\left|\frac{du_r}{dr} + \frac{u_r}{r}\right|^2 dr,$$

$$I_2 = k^2 \int r\left|u_r\right|^2 dr,$$

$$I_3 = k^2 \int r\left[\frac{N^2}{|p|^2} + \frac{\phi(r)}{|p|^2} + \frac{2\Omega_A^2}{k^2|p|^2 r^2}\right]\left|u_r\right|^2 dr,$$

$$I_4 = -k^2 \int \frac{rk^*(DN_0)g\tau\left|u_r\right|^2}{\rho_0\left(p + i\frac{DN_0}{N_0}g\tau\right)} dr$$

and

$$I_5 = k^2 \int \frac{rk^*(DN_0)^2(g\tau)^2\left|u_r\right|^2}{N_0\rho_0\left|p + i\frac{DN_0}{N_0}g\tau\right|^2} dr$$

Clearly $I_1, I_2 \text{ and } I_5$ are positive definite integrals and $I_3 \text{ and } I_4$ are also positive definite if

$$\varphi(r) + N^2 + \frac{2\Omega_A^2}{(k_r)^2} > 0 \quad \text{and} \quad DN_0 < 0 \qquad (19a,b)$$

Then under the above conditions, it follows from (18) that $p_i > 0$, implying thereby, the stability of the system.

First condition of equation (19) is quite important in nature and it illustrates the role of various physical quantities. Clearly, condition (19 a) is satisfied if either

(a) $\phi(r) > 0$; recovery of Rayleigh's criterion for $DN_0 < 0$

or

(b) If $\phi(r) < 0$ then for $\phi(r) + N^2(r) > 0$

or

(c) If $\phi(r) + N^2(r) < 0$ then for $\phi(r) + N^2(r) + \frac{2\Omega_A^2}{k^2 r^2} > 0$

condition (b) clearly ensures the stability of the system even when $\phi(r)<0$ and thus a stabilizing role of N^2.

Condition (a) ensures the stability of the system even when $\phi(r) + N^2(r) < 0$ in the presence of a magnetic field when the condition $\phi(r) + N^2(r) + \frac{2\Omega_A^2}{k^2 r^2} > 0$

This establishes a stabilizing role of magnetic field.

(b) **Modified Rayleigh's Criterion for $DN_0>0$**

Equation (18) can also written as

$$p_i\left[I_1 + I_2 + k^2\right] r \left\{N^2 + \phi(r) + \frac{2\Omega_A^2}{k^2 r^2} - \frac{\left(p_r^2 + p_i^2\right)k^* g\tau(DN_0)}{\left[p_r^2 + \left(p_i + \frac{DN_0}{N_0}g\tau\right)^2\right]\rho_0}\right\}\frac{\left|u_r\right|^2}{|p|^2} d = I_5$$

(20)

If the condition

$$DN_0 > 0 \qquad (21)$$

and

$$\left[N^2 + \phi(r) + \frac{2\Omega_A^2}{k^2 r^2} - \frac{\left(p_r^2 + p_i^2\right)k^* g\tau(DN_0)}{\left[p_r^2 + \left(p_i + \frac{DN_0}{N_0}g\tau\right)^2\right]\rho_0}\right] > 0$$

(22)

hold in the range of r then $p_i > 0$ necessarily which ensures the stability of the system.

Since for $DN_0 > 0$, we have

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$$\frac{\left(p_r^2 + p_i^2\right)}{p_r^2 + \left(p_i + \frac{DN_0}{N_0} g\tau\right)} < 1$$

therefore; condition (22) can be replaced by a stronger condition, namely

$$\left[N^2 + \phi(r) + \frac{2\Omega_A^2}{k^2 r^2} - \frac{k^* g\tau\left(DN_0\right)}{\rho_0}\right] > 0 \qquad (23)$$

everywhere in the range of r i.e. $R_1 \le r \le R_2$.

Thus for $DN_0 > 0$, the system is stable if the modified Rayleigh discriminant (23) is positive in the region $R_1 \le r \le R_2$

4 EXISTENCES OF NON-OSCILLATORY MODES

Multiplying (17) by ru_r^* and integrating over the range of r. The real part of the equation is

$$p_r\left[I_1 + I_2 + I_3 + I_4\right] = 0$$

where

$$I_1 = \int r\left|\frac{du_r}{dr} + \frac{u_r}{r}\right|^2 dr,$$

$$I_2 = \mathrm{k}^2 \int r\left|u_r\right|^2 dr,$$

$$I_3 = -\mathrm{k}^2 \int r\left[\phi(r) + N^2 + \frac{2\Omega_A^2}{k^2 r^2}\right]\frac{\left|u_r\right|^2}{\left|p\right|^2} dr$$

and

$$I_4 = k^2 \int \frac{k^*\left(DN_0\right)g\tau\left|u_r\right|^2}{\rho_0\left|p_r + i\left(p_i + \frac{DN_0}{N_0} g\tau\right)\right|^2} dr$$

since I_1, I_2 are positive definite integrals and the integrals I_3 and I_4 are also positive definite if

$$\left[\phi(r) + N^2 + \frac{2\Omega_A^2}{k^2 r^2}\right] < 0 \quad \text{and} \quad DN_0 > 0 \qquad (24\text{ a,b})$$

hold in the range of the integration, it follows that $p_r = 0$ so that the modes are non-oscillatory.

5 STABILITY OF OSCILLATORY MODES

The existence of oscillatory modes is not ruled out. In fact, if either any one of conditions (24 a, b) or both are not satisfied, then oscillatory modes may exist. If this is the case, then we prove below the stability of such oscillatory modes.

Equation (17) can now be rewritten as

$$p^2 D\left[\frac{du_r}{dr} + \frac{u_r}{r}\right] + \mathrm{k}^2 \phi(r) u_r$$

$$-\mathrm{k}^2 p^2\left[1 + \frac{k^*\left(DN_0\right)g\tau}{\rho_0 p\left(p + i\frac{DN_0}{N_0} g\tau\right)} - \frac{N^2}{p^2}\right]u_r + \frac{2\Omega_A^2}{r^2} u_r = 0 \qquad (25)$$

Multiplying this equation by ru_r^* and integrating over the range of r and then the integrating part is

$$p_r\left[2p_i I_1 - k^2 I_2\right] = 0$$

Where

$$I_1 = \int\left[r\left|\frac{du_r}{dr}\right|^2 + \frac{\left|u_r\right|^2}{r} + D\left(\left|u_r\right|^2\right) + k^2 r\left|u_r\right|^2\right] dr,$$

and

$$I_2 = \int \frac{k^*\left(DN_0\right)^2 g^2 \tau^2}{\rho_0 N_0\left|p + i\frac{DN_0}{N_0} g\tau\right|} r\left|u_r\right|^2 dr,$$

Since I_1 and I_2 are definite positive integrals and $p_r \ne 0$ (for oscillatory modes); it follows that $p_i > 0$.

Thus the oscillatory modes, if exist, are stable.

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